Final Shape of a Dynamically Loaded Elastic-Plastic Redundant Structure

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SEVERAL numerical methods[1 2] have been developed for solving for the inelastic response of dynamically loaded beams and rings For the case of a ring, a number of tests using impulsive loads have been performed A limitation of these tests is that little is learned other than the shape before loading and after loading Consequently, the problem of how to calculate the final deformed shape is posed The numerical methods will not give this final shape unless structural damping is included However, if damping is included, the computer time may be prohibitive

A clue to a method of analysis of final shape is presented if one realizes the fact that in order to include elastic-plastic material response in an analysis, the history of material strain must be known That is, at any time T_0 the internal loads are composed of two parts, the vibrating component and the residual component caused by plastic strains The problem is to separate these components after all plastic response has occurred as the residual component determines the final shape

For purposes of this discussion let us consider a ring that has been loaded impulsively and, at time T_0, has entered the period of undamped elastic vibration Known factors at this time are the internal loads M_0 and N_0, the middle surface strain ϵ_0, and curvature κ_0 Note that these average strains and curvatures provide continuity of the ring

Consider now all cross sections of the ring and remove the internal loads An elastic springback will occur and will be given by the equations

$$\epsilon = (-N_0)/EA \tag{1}$$

$$\kappa = (-M_0)/EI \tag{2}$$

The average strain and curvature at each cross section is now

$$\epsilon = \epsilon_0 + \epsilon_s \tag{3}$$

$$\kappa = \kappa_0 + \kappa_s \tag{4}$$

However, continuity of the ring is not provided, and a residual load system must be introduced in order to satisfy continuity If no plastic deformation of the ring has occurred, the strains and curvatures ϵ and κ will be identically zero and continuity of the ring is assured

Standard structure analysis methods may be used to develop the residual loads and strains A convenient method is that of virtual work Consider the statically determinate symmetric ring shown in Fig 1 and the residual loads M_1m_1 and M_2m_2 M_1 and M_2 may be defined by the continuity equation

$$[\alpha_{ij}]\,[M_i] = -[b_i] \tag{5}$$

where

$$a_{ij} = \int_0^\pi m_i m_j \frac{Rd\phi}{EI} + \int_0^\pi n_i n_j \frac{Rd\phi}{EA}$$

$$b_i = \int_0^\pi m_i \kappa R d\phi + \int_0^\pi n_i \epsilon R d\phi$$

$$i,j = 1,2$$

Received January 20, 1964 This work resulted from an investigation sponsored by Lockheed Missiles and Space Company, Sunnyvale, Calif

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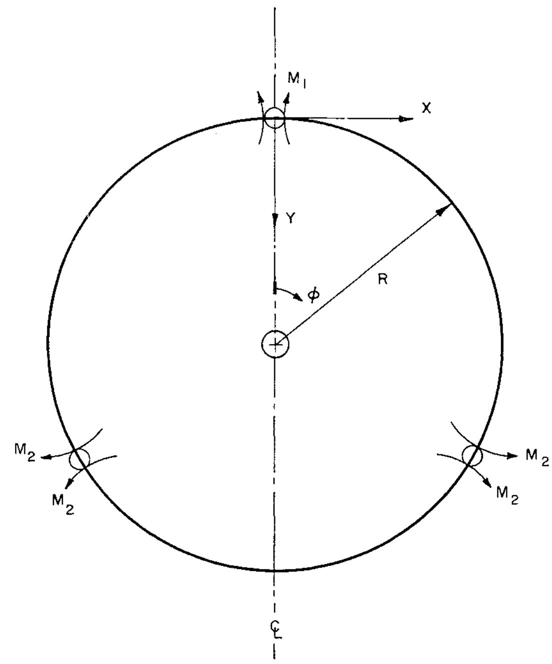

Fig 1 Sketch showing static redundance and coordinates for ring

The moment caused by a unit M_i is m_i, and n_i is the axial force caused by a unit M_i

The final residual loads and strains are

$$M_F = M_1m_1 + M_2m_2 \tag{6}$$

$$N_F = M_1n_1 + M_2n_2 \tag{7}$$

$$\epsilon_F = \epsilon + (N_F/EA) \tag{8}$$

$$\kappa_F = \kappa + (M_F/EI) \tag{9}$$

The final shape of the ring can most easily be determined by a deformation equation commonly used in arch analysis Because a ring is free in space, displacements have no meaning unless a reference is defined Displacements from other reference points are simply obtained by rigid body transformation For this example, the displacements are computed with reference to the displaced top centerline and are given by Eqs (10) and (11):

$$\delta_x = \int_0^\phi \epsilon_F \cos\beta R d\beta - \int_0^\phi \kappa_F(\cos\phi - \cos\beta)R^2 d\beta \tag{10}$$

$$\delta_y = \int_0^\phi \epsilon_F \sin\beta R d\beta - \int_0^\phi \kappa_F(\sin\phi - \sin\beta)R^2 d\beta \tag{11}$$

where the coordinates x and y are indicated in Fig 1

Although the methods of analysis used here are elementary, the novelty in this approach is the application of two independent methods of analysis to obtain a desired end point The initial problem was that of the elastic-plastic dynamic response of an impulsively loaded ring, yet static analysis was a useful tool in defining the end point to the dynamic solution Although thermal strains were not mentioned, the procedure is valid if thermal strains were considered A "room" temperature final shape is obtained by adding the thermal strains to the elements of matrix B of Eq (5)

References

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[2] Ball, R E and Schjelderup, H C , "A feasibility study of the numerical computation of the inelastic response of impulsively loaded shells," National Engineering Science Co Rept P-443-2 (1963)